Calcul des déformations des fils élastiques

Fils élastiques en arc de cercle - Forces distribuées radiales

Forces centrifuges

Fil rond en cuivre

$d := 0.6 \cdot mm$ $\quad S := \pi \cdot \frac{d^2}{4}$ $\quad E := 1.1 \cdot 10^5 \cdot N \cdot mm^{-2}$ $\quad G := \frac{E}{2.6}$ $\quad \rho := 8.9 \cdot 10^3 \cdot kg \cdot m^{-3}$

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$J_t := J_{t\_circ}(d)$ $\quad I_{22} := I_{f\_circ}(d)$ $\quad I_{33} := I_{22}$

$W_t := W_{t\_circ}(d)$ $\quad W'_t := W_t$ $\quad W_{f2} := W_{f\_circ}(d)$ $\quad W_{f3} := W_{f2}$

Caractéristiques de l'arc de cercle $\quad R := 23 \cdot mm$ $\quad \psi_{AB} := 75 \cdot deg$

Forces extérieures en bout d'arc $\quad \psi_F := \psi_{AB}$

$F_x := 0 \cdot N$ $\quad F_y := 0 \cdot N$ $\quad F_z := 0 \cdot N$ $\quad C_x := 0 \cdot N \cdot mm$ $\quad C_y := 0 \cdot N \cdot mm$ $\quad C_z := 0 \cdot N \cdot mm$

Force centrifuge $\quad \Omega := 1300 \cdot \frac{2 \cdot \pi}{60 \cdot s}$ $\quad q_0 := \rho \cdot S \cdot R \cdot \Omega^2$ $\quad q_0 = 1.073\ m^{-1} N$ $\quad \psi_q := \psi_{AB}$

$q_x(\chi) := q_0 \cdot cos(\chi)$ $\quad q_y(\chi) := q_0 \cdot sin(\chi)$ $\quad q_z(\chi) := 0 \cdot N \cdot m^{-1}$

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - F&C&q.mcd(R)

Valeur de tests transitoires $\quad \alpha_m := 20 \cdot deg$

Torseur des forces de cohésion

$\mathbf{M_{cq}}(\psi_F, \psi_q, \alpha_m)^T = (0 \quad 0 \quad -0.242)\ N \cdot mm$

Sollicitations

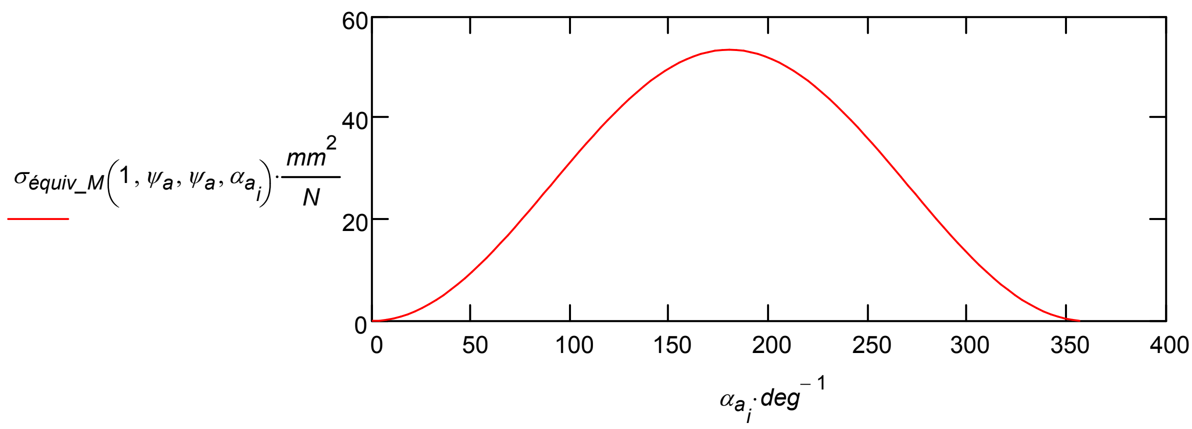
$\mathbf{e'_1}(\alpha_m)^T = (-0.342 \quad 0.94 \quad 0)$ $\quad \mathbf{e'_2}(\alpha_m)^T = (-0.94 \quad -0.342 \quad 0)$ $\quad \mathbf{e'_3}(\alpha_m)^T = (0 \quad 0 \quad 1)$

Moment de torsion $\quad M_t(\psi_F, \psi_q, \alpha_m) = 0\ N \cdot mm$

Moments de flexion $\quad M_{f2}(\psi_F, \psi_q, \alpha_m) = 0\ N \cdot mm$ $\quad M_{f3}(\psi_F, \psi_q, \alpha_m) = -0.242\ N \cdot mm$

Contraintes

Cas d'un anneau fendu $\quad n := 101$ $\quad i := 1 .. n - 1$ $\quad \psi_a := 360 \cdot deg$ $\quad \alpha_{a_i} := (i - 1) \cdot \frac{\psi_a}{n - 1}$



Calcul des déplacements par les intégrales de Mohr

Position du déplacement désiré $\alpha_M := 40 \cdot deg$

Calcul des déplacements linéiques

Déplacement dans la direction de Ox $\lambda := 0 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0.05\,mm$

$\delta_x(\alpha) := \delta_v(\psi_F, \psi_q, \alpha, \lambda, \gamma)$ $\boxed{\delta_x(\alpha_M) = 0.05\,mm}$

Déplacement dans la direction de Oy $\lambda := 90 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0.024\,mm$

$\delta_y(\alpha) := \delta_v(\psi_F, \psi_q, \alpha, \lambda, \gamma)$ $\boxed{\delta_y(\alpha_M) = 0.024\,mm}$

Déplacement dans la direction de *R* $\lambda := \alpha_M$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_F, \psi_q, \alpha_M, \lambda, \gamma) = 0.053\,mm$

$\delta_R(\alpha) := \delta_v(\psi_F, \psi_q, \alpha, \lambda, \gamma)$ $\boxed{\delta_R(\alpha_M) = 0.053\,mm}$

Calcul des déplacements angulaires

Déplacement angulaire autour de l'axe normal au plan de l'arc $\lambda_c := 0 \cdot deg$ $\gamma_c := 0 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv2}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv3}(\psi_F, \psi_q, \alpha_M, \lambda_c, \gamma_c) = -0.327\,deg$

$\theta_z(\alpha) := \theta_{fcv3}(\psi_F, \psi_q, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_z(\alpha_M) = -0.327\,deg}$

Solution analytique

Moment fléchissant $M_{f3}(\psi, \alpha') := q_0 \cdot R^2 \cdot (\sin(\alpha') \cdot \sin(\psi) + \cos(\alpha') \cdot \cos(\psi) - 1)$

Déplacement selon Ox $M_v(\alpha, \alpha') := -R \cdot (\sin(\alpha) - \sin(\alpha'))$

$$\delta_{q1}(\psi, \alpha) := \frac{R}{E \cdot I_{33}} \cdot \int_0^{\alpha} M_{f3}(\psi, \alpha') \cdot M_v(\alpha, \alpha')\, d\alpha' \qquad \delta_{q1}(\psi_F, \alpha_M) = 0.05\,mm$$

$$\delta_{q1}(\psi, \alpha) := \frac{-q_0 \cdot R^4}{2 \cdot E \cdot I_{33}} \cdot [\sin(\alpha) \cdot (\cos(\psi) \cdot \sin(\alpha) + 2 \cdot \sin(\psi) - \cos(\alpha) \cdot \sin(\psi) - 2 \cdot \alpha) - \sin(\psi) \cdot \alpha + 2 \cdot (1 - \cos(\alpha))]$$

Déplacement selon Oy $M_v(\alpha, \alpha') := -R \cdot (-\cos(\alpha) + \cos(\alpha'))$

$$\delta_{q2}(\psi, \alpha) := \frac{R}{E \cdot I_{33}} \cdot \int_0^{\alpha} M_{f3}(\psi, \alpha') \cdot M_v(\alpha, \alpha')\, d\alpha' \qquad \delta_{q2}(\psi_F, \alpha_M) = 0.024\,mm$$

$$I(\psi, \alpha) := [\cos(\alpha) \cdot [-2 \cdot \sin(\psi) \cdot (1 - \cos(\alpha)) - \sin(\alpha) \cdot \cos(\psi) + 2 \cdot \alpha] + \sin(\alpha) \cdot (\sin(\alpha) \cdot \sin(\psi) - 2) + \cos(\psi) \cdot \alpha]$$

$$\delta_{q2}(\psi, \alpha) := \frac{-q_0 \cdot R^4}{2 \cdot E \cdot I_{33}} \cdot I(\psi, \alpha)$$

Déplacement angulaire $M_v(\alpha, \alpha') := 1$

$$\theta_{q3}(\psi, \alpha) := \frac{R}{E \cdot I_{33}} \cdot \int_0^{\alpha} M_{f3}(\psi, \alpha') \cdot M_v(\alpha, \alpha') \, d\alpha' \qquad \theta_{q3}(\psi_F, \alpha_M) = -0.327\,deg$$

$$\theta_{q3}(\psi, \alpha) := \frac{q_0 \cdot R^3}{E \cdot I_{33}} \cdot [(1 - cos(\alpha)) \cdot sin(\psi) + sin(\alpha) \cdot cos(\psi) - \alpha]$$

Déplacements cartésiens totaux en M

$\alpha_M = 40\,deg$

$$\Delta_q(\psi, \alpha) := \begin{pmatrix} \delta_{q1}(\psi, \alpha) \cdot m^{-1} \\ \delta_{q2}(\psi, \alpha) \cdot m^{-1} \\ \theta_{q3}(\psi, \alpha) \end{pmatrix} \qquad \Delta_q(\psi_F, \alpha_M) = \begin{pmatrix} 0.05 \\ 0.024 \\ -5.703 \end{pmatrix} 10^{-3}$$

Cas particuliers

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Définition Atan.mcd(R)

Quart de cercle

$\psi_{AB} := 90 \cdot deg$

$L := R \cdot \psi_{AB} \qquad L = 36.128\,mm$

$$\Delta_{q90} := \frac{q_0 \cdot R^3}{E \cdot I_{33}} \cdot \begin{bmatrix} \left(\frac{3 \cdot \pi}{4} - 2\right) \cdot R \\ \frac{1}{2} \cdot R \\ \left(1 - \frac{\pi}{2}\right) \cdot m \end{bmatrix} \cdot \frac{1}{m} \qquad \Delta_{q90} = \begin{pmatrix} 0.153 \\ 0.214 \\ -10.645 \end{pmatrix} 10^{-3}$$

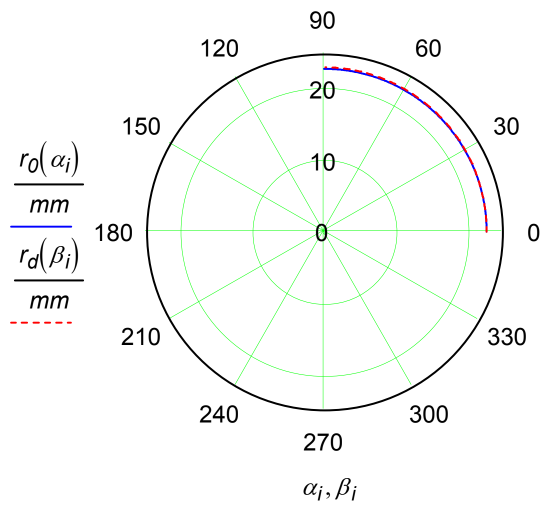
$\delta_{q1}(\psi_{AB}, \psi_{AB}) = 0.153\,mm \qquad \delta_{q2}(\psi_{AB}, \psi_{AB}) = 0.214\,mm \qquad \theta_{q3}(\psi_{AB}, \psi_{AB}) = -0.61\,deg$

Graphe de la déformation

$n := 201 \qquad i := 1 .. n \qquad \alpha_i := \frac{\psi_{AB}}{n - 1} \cdot (i - 1) \qquad x_0(\alpha) := R \cdot cos(\alpha) \qquad y_0(\alpha) := R \cdot sin(\alpha) \qquad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_{q1}(\psi_{AB}, \alpha) \qquad y_d(\alpha) := y_0(\alpha) + \delta_{q2}(\psi_{AB}, \alpha) \qquad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i)) \qquad \beta_1 = 0\,deg \qquad \beta_n = 89.623\,deg$



$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$

$\alpha_0 := 0 \qquad \alpha_{max} := \psi_{AB}$

$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2} \, d\alpha$$

$L_d = 36.13\,mm \qquad L = 36.128\,mm$

Demi-cercle

$\psi_{AB} := 180 \cdot deg$

$L := R \cdot \psi_{AB}$ $\quad L = 72.257\,mm$

$$\Delta_{q180} := \frac{q_0 \cdot R^3}{E \cdot I_{33}} \cdot \begin{pmatrix} -2 \cdot R \\ \frac{3 \cdot \pi}{2} \cdot R \\ -\pi \cdot m \end{pmatrix} \cdot \frac{1}{m} \qquad \Delta_{q180} = \begin{pmatrix} -0.858 \\ 2.021 \\ -58.59 \end{pmatrix} 10^{-3}$$

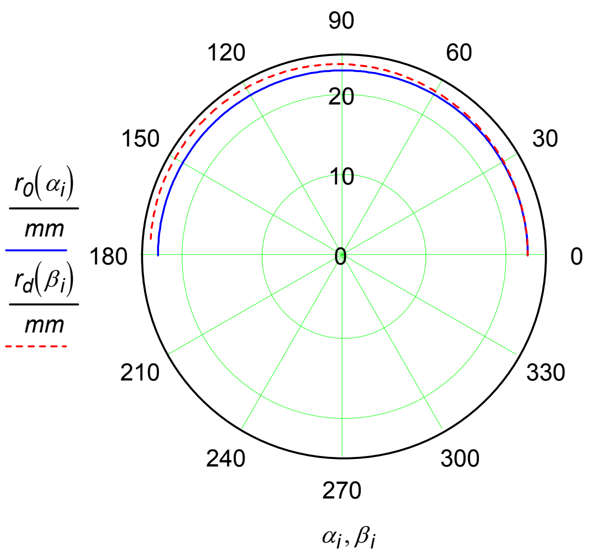
$\delta_{q1}(\psi_{AB}, \psi_{AB}) = -0.858\,mm$ $\quad \delta_{q2}(\psi_{AB}, \psi_{AB}) = 2.021\,mm$ $\quad \theta_{q3}(\psi_{AB}, \psi_{AB}) = -3.357\,deg$

Graphe de la déformation

$n := 201$ $\quad i := 1 .. n$ $\quad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1)$ $\quad x_0(\alpha) := R \cdot \cos(\alpha)$ $\quad y_0(\alpha) := R \cdot \sin(\alpha)$ $\quad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_{q1}(\psi_{AB}, \alpha)$ $\quad y_d(\alpha) := y_0(\alpha) + \delta_{q2}(\psi_{AB}, \alpha)$ $\quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\quad \beta_1 = 0\,deg$ $\quad \beta_n = 175.157\,deg$



$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha)$ $\quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$

$\alpha_0 := 0$ $\quad \alpha_{max} := \psi_{AB}$

$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$$

$L_d = 72.329\,mm$ $\quad L = 72.257\,mm$

Anneau fendu

$\psi_{AB} := 360 \cdot deg$

$L := R \cdot \psi_{AB}$ $\quad L = 144.513\,mm$

$$\Delta_{q360} := \frac{q_0 \cdot R^3}{E \cdot I_{33}} \cdot \begin{pmatrix} 0 \cdot R \\ -3 \cdot \pi \cdot R \\ 2 \cdot \pi \cdot m \end{pmatrix} \cdot \frac{1}{m} \qquad \Delta_{q360} = \begin{pmatrix} 0 \\ -4.043 \\ 117.179 \end{pmatrix} 10^{-3}$$

$\delta_{q1}(\psi_{AB}, \psi_{AB}) = 0\,mm$ $\quad \delta_{q2}(\psi_{AB}, \psi_{AB}) = -4.043\,mm$ $\quad \theta_{q3}(\psi_{AB}, \psi_{AB}) = -6.714\,deg$

Graphe de la déformation

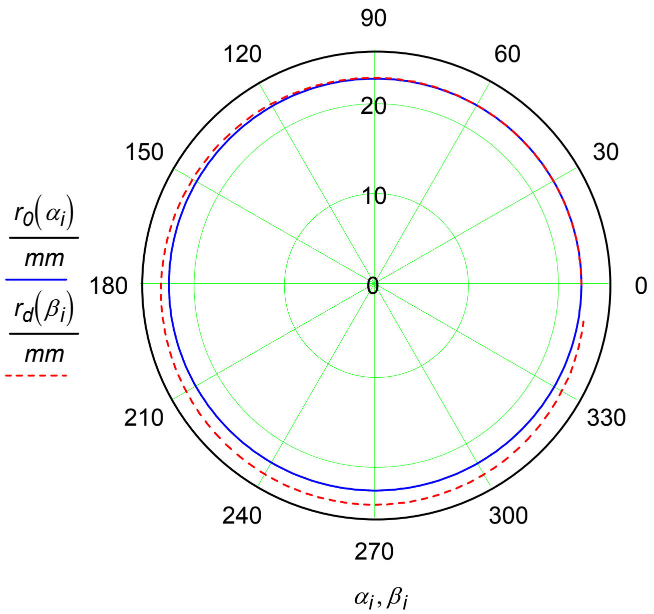
$n := 201$ $\quad i := 1 .. n$ $\quad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1)$ $\quad x_0(\alpha) := R \cdot \cos(\alpha)$ $\quad y_0(\alpha) := R \cdot \sin(\alpha)$ $\quad r_0(\alpha) := \sqrt{x_0(\alpha)^2 + y_0(\alpha)^2}$

$x_d(\alpha) := x_0(\alpha) + \delta_{q1}(\psi_{AB}, \alpha)$ $\quad y_d(\alpha) := y_0(\alpha) + \delta_{q2}(\psi_{AB}, \alpha)$ $\quad r_d(\alpha) := \sqrt{x_d(\alpha)^2 + y_d(\alpha)^2}$

$\beta_i := Atan(x_d(\alpha_i), y_d(\alpha_i))$ $\quad \beta_1 = 0\,deg$ $\quad \beta_n = 350.031\,deg$

$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha)$ $\quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha)$ $\quad \alpha_0 := 0$ $\quad \alpha_{max} := \psi_{AB}$

Fils en arc de cercle
Forces distribuées radiales



$$L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2}\, d\alpha$$

$L_d = 144.906\,mm$ $\qquad$ $L = 144.513\,mm$